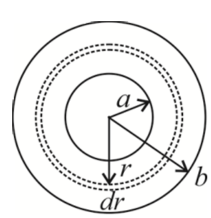
Solutions to JEE Advanced Booster Test – 3|JEE 2024

PHYSICS

SECTION-1

Single Choice Type

1.(A) Suppose that the inner sphere is at a higher potential than outer sphere. Let the current be i.
Consider a thin shell of thickness dr at a distance r from the centre. Let voltage across it be dV. Then, applying $V = iR$



For thin shell: $dV = i \cdot \dfrac{dr}{\sigma 4\pi r^2} \Rightarrow Edr = i \cdot \dfrac{dr}{4\pi r^2} \cdot \dfrac{1}{kE}$

$\therefore \quad E = \sqrt{\dfrac{i}{4\pi k r^2}} = \dfrac{C}{r}$ where $C = \sqrt{\dfrac{i}{4\pi k}}$

Using, $E = -\dfrac{dV}{dr}$

$$\frac{C}{r} = -\frac{dV}{dr} \Rightarrow C(\ell n r)_a^b = V_a - V_b$$

$$V = V_a - V_b = C\ln(b/a)$$

$$V = \sqrt{\frac{i}{4\pi k}}\ell n(b/a) \Rightarrow i = \frac{V^2 4\pi k}{[\ell np(b/a)]^2}$$

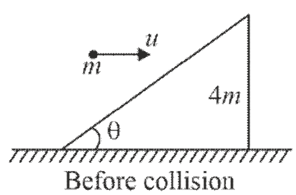
2.(D) $B = \dfrac{\mu_0 I}{4\pi(r_1)}(\sin\theta_2 - \sin\theta_1)$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi(r\sin\alpha)}\left[\sin\frac{\pi}{2} - \sin\left(\frac{\pi}{2} - \alpha\right)\right]$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi(r\sin\alpha)}(1-\cos\alpha)$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi r\left(2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)\right)}\left(2\sin^2\left(\frac{\alpha}{2}\right)\right) = \frac{\mu_0 I}{4\pi r}\tan\left(\frac{\alpha}{2}\right)$$

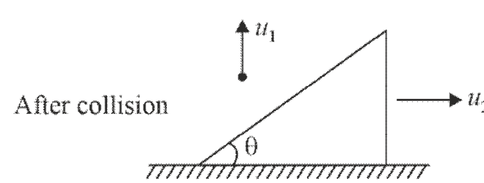
3.(A) Parallel to inclined plane, $u\cos\theta = v_1\sin\theta \Rightarrow v_1 = \dfrac{u}{\sqrt{3}}$



Along horizontal $mu = 4mv_2 \Rightarrow v_2 = \dfrac{u}{4}$

Along common normal $v_2\sin\theta + v_1\cos\theta = eu\sin\theta$

$$\Rightarrow \frac{u}{4}.\frac{\sqrt{3}}{2} + \frac{u}{\sqrt{3}}\frac{1}{2} = eu\frac{\sqrt{3}}{2} \Rightarrow e = \frac{7}{12}$$



4.(B) $mg\frac{L}{2}=\frac{1}{3}mL^2\alpha$

$\therefore\ \alpha=\frac{3g}{2L}$ and $\alpha x=g \Rightarrow \frac{3g}{2L}x=g \quad \therefore x=\frac{2L}{3} \qquad \therefore$ Distance from $B=\frac{L}{3}$

5.(D) $T=\frac{2\pi m}{qB}=\frac{2\pi\times 1}{1\times 1}=2\pi s$; $R=\frac{mv}{qB}=\frac{1\times 1}{1\times 1}=1m$

x = 0, $y=\frac{1}{2}\frac{E_y q}{M}t^2=\frac{1}{2}\frac{1\times 1}{1}\times\pi^2=\frac{\pi^2}{2}m$

z = 2 R = 2m ; Co-ordinates will be $\left(0,\frac{\pi^2}{2},2\right)m$

Multiple Correct Type

6.(ABD) $\because \quad Q_0=C_0V \qquad \because \quad \frac{Q_0}{2}=C_1V'=C_2V'$

$C_1=\frac{\epsilon_0 KA/3}{d}=\frac{KCO_0}{3}$; $C_2=\frac{\epsilon_0 2A/3}{d}=\frac{2C_0}{3}$

$\because \quad C_1=C_2 \Rightarrow k=2 \Rightarrow C_1=C_2=\frac{2C_0}{3}$

New potential difference $=\frac{Q_0\times 3}{2\times 2C_0}=\frac{3V}{4}$

Charge on dielectric slap = $Q_{bound}=Q_{free}\left(1-\frac{1}{K}\right)=\frac{Q_0}{2}\left(1-\frac{1}{2}\right)=\frac{Q_0}{4}=\frac{C_0V}{4}$

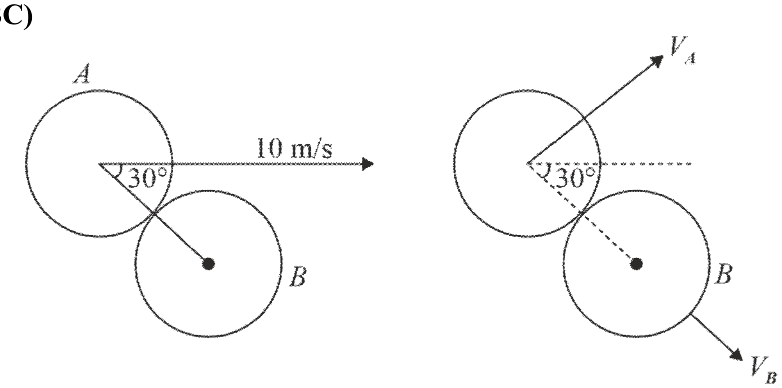
Force before insertion $=\frac{Q_0}{2A\epsilon_0}.Q_0=\frac{Q_0^2}{2A\epsilon_0}$

Force after insertion : $F_1=F_{\text{covered}}=\frac{Q_0/2}{2\epsilon_0 .A/3},\frac{Q_0}{2}=\frac{3}{8}\frac{Q_0{}^2}{\epsilon_0 A}$

$$F_2=F_{un\text{covered}}=\frac{Q_0/2}{2\epsilon_0 .2A/3}.\frac{Q_0}{2}=\frac{3}{16}\frac{Q_0{}^2}{\epsilon_0 A}$$

$$F_{net}=F_1+F_2=\frac{9}{16}\frac{Q_0^2}{\epsilon_0 A}\ ;\ \frac{F_{before}}{F_{atter}}=\frac{1/2}{9/16}=\frac{8}{9}$$

7.(ABC)



$v_B=10\cos 30°=5\sqrt{3}$ and $v_A=10\sin 30°=5$

8.(AD) $\vec{B}$ due to body diagonal $= \vec{0}$

$\vec{B}$ due to edge $\dfrac{\mu_0 i}{4\pi(a/\sqrt{2})}\left(\sqrt{\dfrac{1}{3}}+\sqrt{\dfrac{1}{3}}\right)$ along $\dfrac{-\hat{i}-\hat{j}}{\sqrt{2}}$ (using the result $\dfrac{\mu_0 i}{4\pi r}(\sin\alpha+\sin\beta)$)

$\vec{B}$ due to face diagonal $\dfrac{\mu_0 i}{4\pi(a/2)}\left(\sqrt{\dfrac{2}{3}}+\sqrt{\dfrac{2}{3}}\right)$ and $\dfrac{-\hat{i}-\hat{j}}{\sqrt{2}}$

Also, magnetic moment $\vec{\mu} = i\vec{A}$.

9.(ABC) $2i_1+3(i_1+i_2-i_3)+4(i_1+i_2)-10=6$

$2i_1 - i_2 = 3$

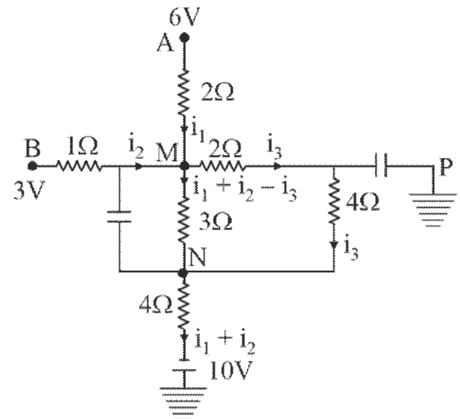
$3(i_1+i_2-i_3)-6i_3=0$

$i_1 = 1.7; i_2 = 0.4; i_3 = 0.7$

$V_M = 2.6\ Volt$

$V_N = -1.6\ Volt$

$Q_{across\ 2\mu f} = 8.4\mu C$



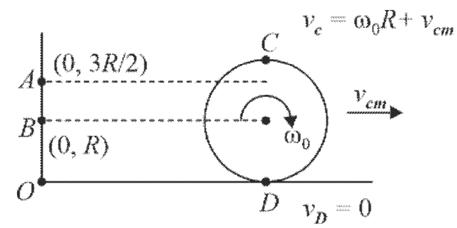
10.(ABD) $L = mv \times R + I\omega$

$L_C = mv_{com}R(-\hat{k}) + I_{com}\omega_0(-\hat{k})$

$L_D = +mv_{com}R(-\hat{k}) + I_{com}\omega_0(-\hat{k})$

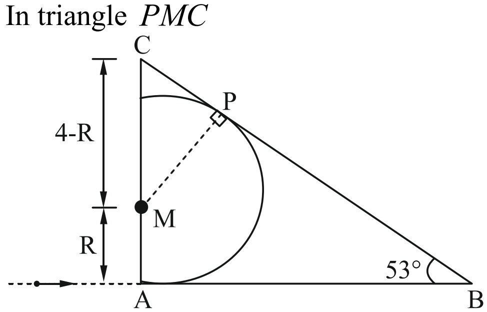
$L_C = L_D$

$L_0 = +mv_{com}R(-\hat{k}) + I_{com}\omega_0(-\hat{k})$



PARAGRAPH FOR Q-11 & 12

11.(C) In triangle PMC

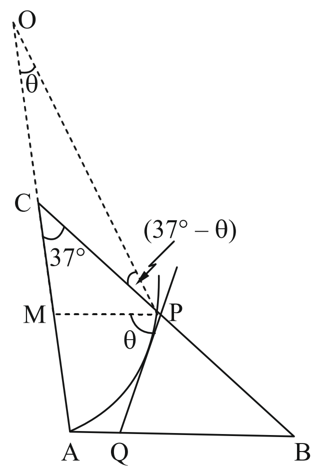


$\cos 53° = \dfrac{MP}{MC}$; $\dfrac{3}{5} = \dfrac{R}{4-R}$; $12 = 8R$

$R = \dfrac{3}{2}m$ (R is the maximum radius of half – circle)

$R_{max} = \dfrac{mu_{max}}{qB} \Rightarrow u_{max} = 3\,m/s$

12.(B) $R=\dfrac{mu}{qB}=24m$



Let, $\angle MPQ=\theta$

By geometry

$\angle CPO=(37-\theta)$

In ΔCPO

$$\frac{OC}{\sin(\angle CPO)}=\frac{OP}{\sin(\angle PCO)}$$

$$\frac{20}{\sin(37°-\theta)}=\frac{24}{\sin(180°-37°)}$$

$$\Rightarrow \quad \frac{5}{\sin(37°-\theta)}=\frac{5\times 6}{3} \Rightarrow \sin(37°-\theta)=\frac{1}{2}$$

$$\theta=\frac{7\pi}{180}rad. \quad \Rightarrow \quad \omega=\frac{qB}{m} \Rightarrow \omega=2rad/\sec. \quad \Rightarrow \quad t=\frac{7\pi}{360}\sec.$$

SECTION-2

1.(–6) $(1+3)v=(1)(8)+(3)(4)=20; v=5m/\sec$

For block A, $\quad W_f=\dfrac{1}{2}(1)(5^2-8^2)=-\dfrac{39}{2}J$

For block B, $\quad W_f=\dfrac{1}{2}(3)(5^2-4^2)=+\dfrac{27}{2}J$

Net work done by friction $=-6J$

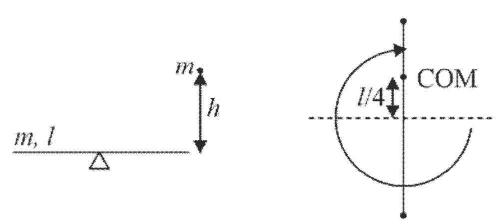
2.(1) Current in circuit

I = $\dfrac{1}{5}A$ = 0.2 A which will pass through 10Ω and 20Ω in both the cases.

$V_1=V_2=\dfrac{1}{2}(30\times 0.2)=3V$; $V_1'=10\times 0.2=2V, V_2'=20\times 0.2=4V$

3.(8) For critical case

Initial KE = P.E. at max height



$2mg\dfrac{l}{4}=\dfrac{1}{2}I_{Hinge}\omega^2$

Angular momentum conservation about hinge

$mv\dfrac{l}{2}=I\omega$

$$m\sqrt{2gh}\frac{l}{2}=\left[\frac{ml^2}{12}+\frac{ml^2}{4}\right]\omega \qquad (v=\sqrt{2gh} \qquad \because v^2-u^2=2gh)$$

$$\frac{6}{4l}\sqrt{2gh}=\omega \quad ; \qquad 2mg\frac{l}{4}=\frac{1}{2}\left[\frac{ml^2}{12}+\frac{ml^2}{4}\right]\left[\frac{36}{16l^2}\times 2gh\right]; \qquad \frac{2l}{3}=h=\frac{2}{3}\times 12=8 \quad ; \quad h=8$$

4.(5) Velocity of first block before collision

$v_1^2 = 1^2 - 2(2)\times 0.16 = 1 - 0.64; v_1 = 0.6 m/s$

By conservation of momentum ; $2\times 0.6 = 2v_1 + 4v_2$

Also $v_2 - v_1 = 0.6$ for elastic collision. It gives $v_2 = 0.4\ m/s$; $v_1 = -0.2\ m/s$

Now distance moved after collision $s_2 = \frac{(0.4)^2}{2\times 2}$ and $s_1 = \frac{(0.2)^2}{2\times 2}; s = s_1 + s_2 = 0.05m = 5cm$.

5.(1.50) During charging for $\tau_1 = R_{eq}C$

$R_{eq} = R_3 = R$

$\tau_1 = RC$

During discharge $= \tau_2 = R_{eq}C$

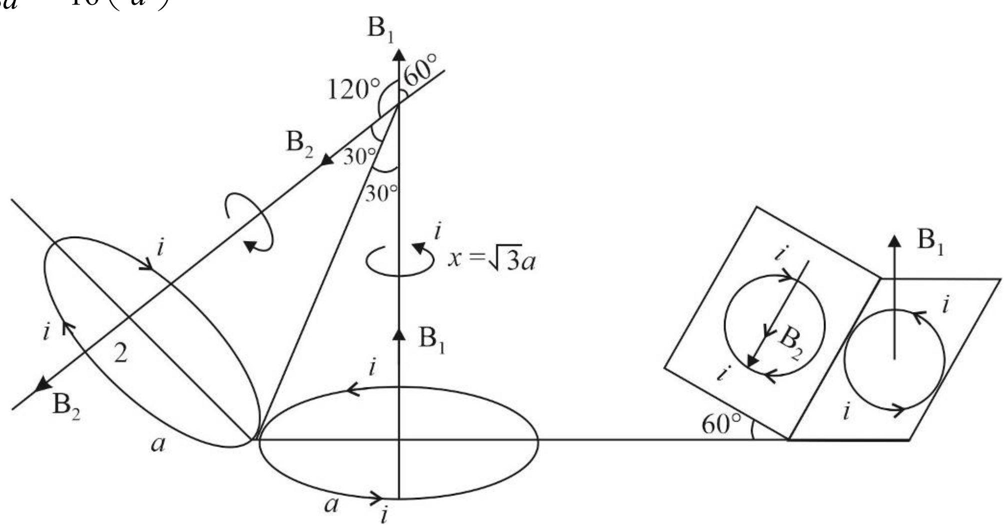
$\text{Req} = \frac{R_1R_2}{R_1 + R_2} + R_3 = \frac{3R}{2}$

$\tau_2 = \frac{3}{2}RC$

$\frac{\tau_2}{\tau_1} = \frac{3}{2} = 1.5$

6.(4) $B_1 = B_2 = \frac{\mu_0 i a^2}{2\left(a^2 + x^2\right)^{3/2}} = \frac{\mu_0 i a^2}{2\left(a^2 + 3a^2\right)^{3/2}}$

$= \frac{\mu_0 i a^2}{2\times 8a^3} = \frac{\mu}{16}\left(\frac{16}{a}\right)\times 10^7$



$B_1 = B_2 = 4\pi\times 10^{-7}\times\frac{1}{\pi}\times 10^7 = 4J$; $B_{net} = 2B_1\cos 60° = B_1 = 4T$

CHEMISTRY

SECTION-1

Single Choice Type

1.(A) $Ag^{+} + 2NH_3 \rightleftharpoons [Ag(NH_3)_2]^{+} \; K_f = 10^8$

For beaker A

$$K_f = \frac{[Ag(NH_3)_2]^{+}}{[Ag^{+}][NH_3]^2}$$

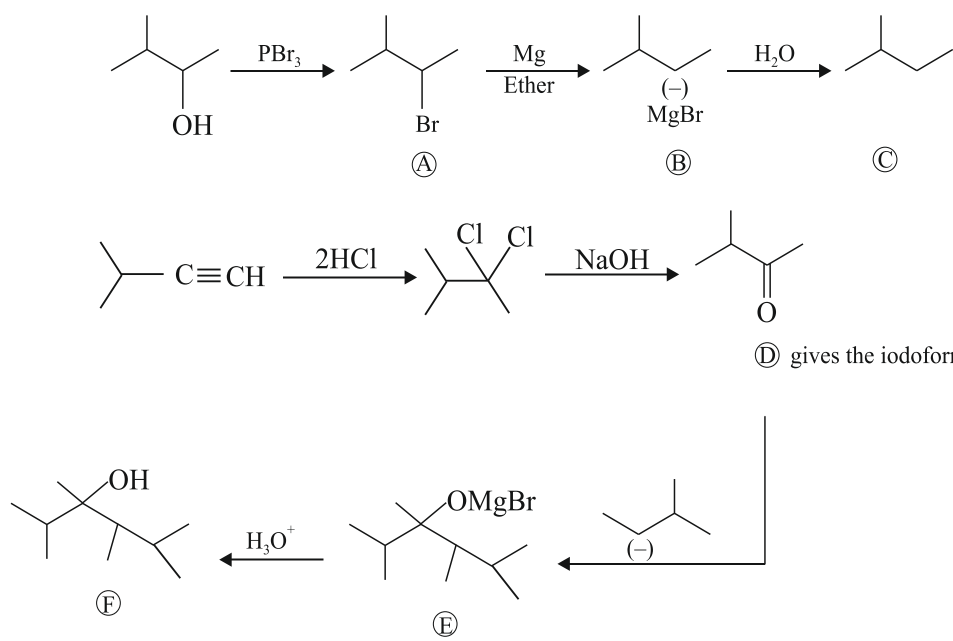
$$[Ag^{+}]_A = \frac{0.1}{K_f(0.1)^2} = \frac{10}{K_f}$$

For Beaker B

$$K_f = \frac{[Ag(NH_3)_2]^{+}}{[Ag^{+}][NH_3]^2} \; ; \; K_f = \frac{(0.02)}{[Ag^{+}](0.2)^2} \; ; \; [Ag^{+}]_B = \frac{1}{2K_f}$$

$$E_{cell} = 0 - \frac{0.06}{1}\log\frac{[Ag^{+}]_B}{[Ag^{+}]_A} = -0.06\log\frac{1}{20} = 0.06\log 20 = 0.06(1+0.3) = 1.3\times 0.06 = 7.8\times 10^{-2}\text{ V}$$

2.(D)



Ⓓ gives the iodoform test

2,3,4,5 - tetramethylhexan-3-ol

3.(B) Fact Based

4.(A) Statements I, II and IV are correct.

At Y, $\frac{dZ}{dP} \to 0$ where attractive forces dominate over repulsive forces.

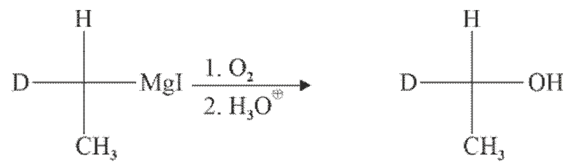
At Y, Potential Energy is minimum.

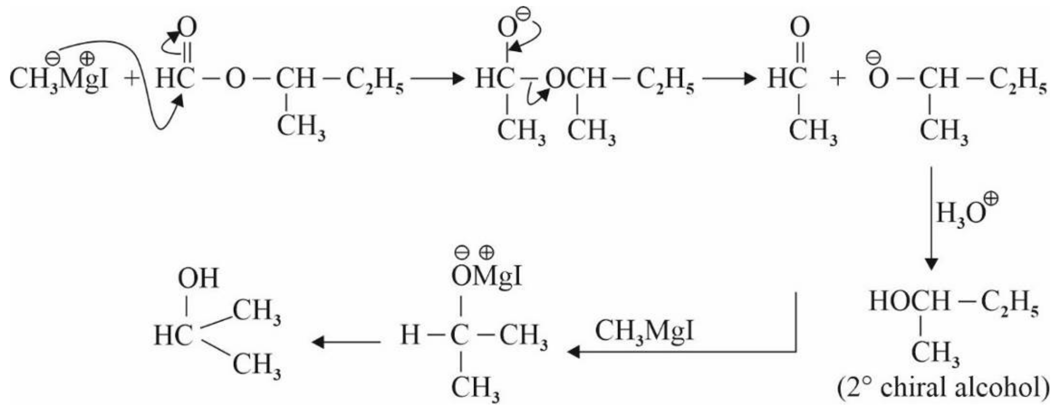
5.(A) Statements (a), (b) & (c) are correct.

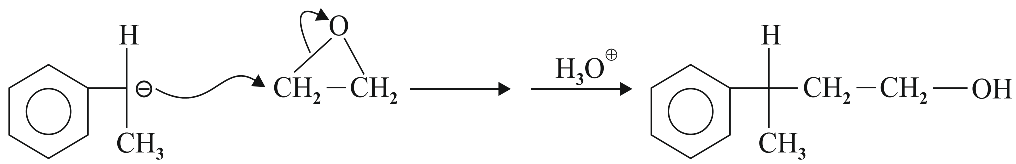
Bond order of N_2^{+} is less than that of N_2 therefore N–N bond strength decreases.

Multiple Correct Type

6.(ACD)

(A) 

(B) 

(C) 

(D) $CH_3-C\equiv N+C_2H_5MgBr \longrightarrow \xrightarrow{H_3O^{\oplus}} CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-C_2H_5$

7.(BCD) $\Delta G^\circ = -nF\,E^o_{cell}$

$$2\times -240\times 10^3 = -4\times 96500\times E^\circ_{cell}$$

$$E^\circ_{cell} = 1.24\,V$$

No. of moles of H_2 needed to produce 240 kJ of useful work = 1

$\therefore$ For 4.8 kJ of useful work,

$$\eta_{H_2} = \frac{4.8}{240} = 0.02$$

$$\eta = \frac{|\Delta G|}{|\Delta H|}\times 100 = \frac{240}{285}\times 100 = 84\%$$

8.(BD) Statements B and D are correct.
Statements A and C are incorrect.

9. (AC)
(A) For Micelle formation, Entropy change is positive.
(B) Micelles formation will take place above T_k and above CMC
(C) $Fe(OH)_3$ sol prepared by the hydrolysis of $FeCl_3$ solution adsorbs Fe^{3+} and this is positively charged.

$$FeCl_3 + 3H_2O \rightleftharpoons Fe(OH)_3 + 3HCl;\quad Fe(OH)_3 + FeCl_3 \rightarrow \underset{\text{Fixed part}}{Fe(OH)_3\,|\,Fe^{3+}} : \underset{\text{Diffused part}}{3Cl^-}$$

(D) Fe^{3+} ions will have greater flocculation power, so smaller flocculating value.

10.(ABD) For ideal gas

$PM = d\,R\,T$

$$M = \frac{d}{P}\ R\,T$$

$\frac{d}{P}$ vs T has a negative slope.

Gas A is non-ideal gas

Gas B is ideal gas.

For B

$M = 3.30 \times 0.082 \times 300 = 81.18\ g\ mol^{-1}$

PARAGRAPH FOR Q-11 & 12

11.(C) $\lambda_m^C = \lambda_m^\infty - b\sqrt{c}$

When $c_1 = 4\times10^{-4}$ $\qquad \lambda_m^c = 107$

And when $c_2 = 9\times10^{-4}$ $\qquad \lambda_m^c = 97$

So $107 = \lambda_m^\infty - b\times 2\times10^{-2}$ $\qquad$ ...(1)

$97 = \lambda_m^\infty - b\times3\times10^{-2}$ $\qquad$ ...(2)

$b = 1000$

$\lambda_m = \lambda_m^\infty - b\sqrt{C}$

$\lambda_m^\infty = \lambda_m + b\sqrt{C} = 107 + 10^3 \times 2\times10^{-2}$

$\lambda_m^\infty = 127\,\text{ohm}^{-1}\,\text{cm}^2\,\text{mol}^{-1}$

12.(D) For 25×10^{-4} (M) NaCl solution

$\lambda_m = \lambda_m^\infty - b\sqrt{C}$

$\lambda_m = 127 - 10^3(25\times10^{-4})^{1/2}$

$\lambda_m = 127 - 10^3 \times 5\times10^{-2}$

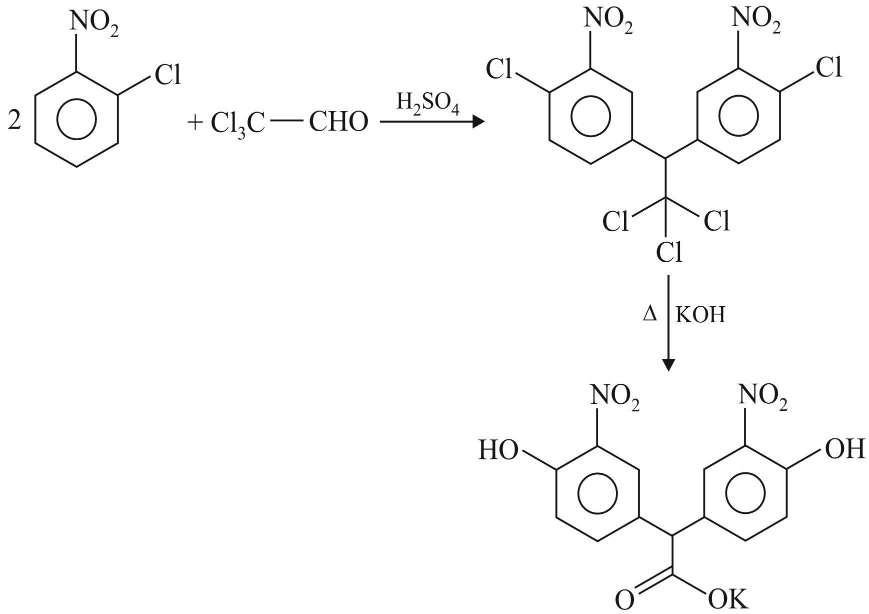
$\lambda_m = 77$

$$\lambda = \frac{1000.G.\sigma}{M}$$

So, $\sigma = \dfrac{M\lambda}{1000\,G} = \dfrac{25\times10^{-4}\times77\times1000}{1000} = 0.1925\,\text{cm}^{-1}$

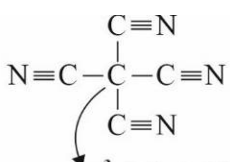
SECTION-2

1.(8)



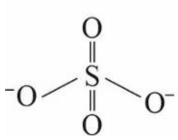
2.(8) VII & X are incorrect.

3.(8)



sp^3 (4 hybrid orbitals having 25% s character)

X = 4



All four S – O bonds are equal due to resonance

Y = 4

X + Y = 4 + 4 = 8

4.(450) $\sqrt{\dfrac{2RT}{M_{SO_2}}} = \sqrt{\dfrac{3R\times T_1}{M_{O_2}}} \Rightarrow \dfrac{2\times T}{64} = \dfrac{3\times 300}{32}$

T = 900K

$$\left(P+\frac{an^2}{V^2}\right)V = nRT$$

$$\left(0.1+\frac{1000\times(0.02)^2}{V^2}\right)V = 20\times 0.02 = 0.1V^2-0.4V+0.4=0 = V^2-4V+4=0$$

$\Rightarrow$ V = 2 L

$$Z=\frac{PV}{nRT}=\frac{0.1\times 2}{20\times 0.02}=0.5$$

X = 900 & Y = 0.5

5.(70) $5.2 = \frac{52}{3} \times \frac{9.65}{96500} \times t(\text{sec.}),$

t(sec) = 3000

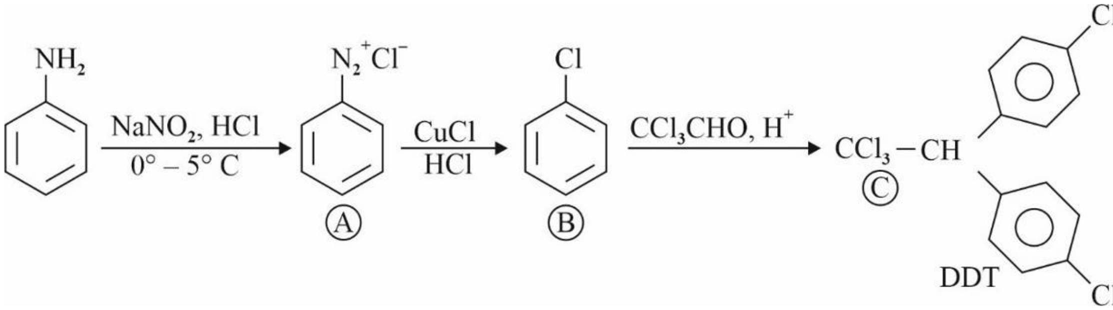
t(min) = 50

1 mole Mn_3O_4 lose $\left(6 - \frac{8}{3}\right) \times 3 = 10$ mole e^-

So, total charge required = 2 × 10 ⇒ 20F

X = 50 & Y = 20

6.(230)



X = 112.5

Y = 5

2X + Y = (2 × 112.5) + 5 = 230

MATHEMATICS

SECTION-1

Single Choice Type

1.(C) $s^2 = at^2 + 2bt + c$

Differentiating w.r.t. t

$$2s\frac{ds}{dt} = 2at + 2b$$

$$s^2.\left(\frac{ds}{dt}\right)^2 = (at+b)^2 = a^2t^2 + 2abt + b^2$$

$$s^2.\left(\frac{ds}{dt}\right)^2 = as^2 + b^2 - ac \; ; \; \left(\frac{ds}{dt}\right)^2 = a + \frac{b^2 - ac}{s^2}$$

Differentiating w.r.t. t again

$$2\frac{ds}{dt}.\frac{d^2s}{dt^2} = 0 - 2\frac{b^2 - ac}{s^3}\frac{ds}{dt} \quad \Rightarrow \frac{d^2s}{dt^2} = \frac{ac - b^2}{s^3}$$

2.(A) Put $x = 8t^3$ then the value of limit

$$L = \lim_{x \to \infty} \frac{f^{-1}(8x) - f^{-1}(x)}{x^{1/3}} = \lim_{t \to \infty} \frac{f^{-1}(64t^3) - f^{-1}(8t^3)}{2t}$$

as $x \to \infty$ $\qquad f(x) \cong 8x^3 \Rightarrow f^{-1}(8x^3) \cong x$

$$\Rightarrow \quad L = \lim_{t \to \infty} \frac{2t - t}{2t} = \frac{1}{2}$$

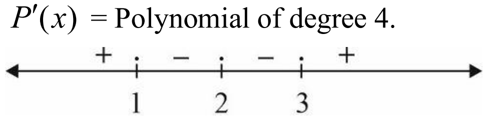
3.(C) Value of limit $L = \lim_{n \to \infty} \dfrac{2\left(\frac{n^4}{4} + ...\right)\left(\frac{n^6}{6}.....\right) - \left(\frac{n^3}{3}.....\right)\left(\frac{n^7}{7}.....\right)}{\left(\frac{n^5}{5}.....\right)^2} = \dfrac{\frac{2}{24} - \frac{1}{21}}{\frac{1}{25}} = \dfrac{25}{28}$

4.(A) $\because \dfrac{\sin\theta}{\theta}$ and $\dfrac{\theta}{\tan\theta}$ both are decreasing functions in $\theta \in \left(0, \dfrac{\pi}{2}\right)$

$$\Rightarrow \quad \text{maxima} = \lim_{\theta \to 0}\left(\frac{\sin\theta}{\theta} + \frac{\theta}{\tan\theta}\right) = 2$$

$$\text{minima} = \lim_{\theta \to \frac{\pi}{2}}\left(\frac{\sin\theta}{\theta} + \frac{\theta}{\tan\theta}\right) = \frac{2}{\pi}$$

5.(B) $P'(x)$ = Polynomial of degree 4.



$\Rightarrow \; P'(x) = 5(x-1)(x-2)^2(x-3)$

Multiple Correct Type

6.(BC) $f'(x)=\lim\limits_{h\to 0}\left(\dfrac{f(x+h)-f(x)}{h}\right)=\lim\limits_{h\to 0}\left(\dfrac{kxh+2h^2}{h}\right)$

$\Rightarrow f'(x)=kx \Rightarrow f(x)=\dfrac{k}{2}x^2+c \;\Rightarrow f(x)=2x^2$ as $k=4, c=0$, as $f(1)=2, f(2)=8$

So, $\quad f(x+y).f\left(\dfrac{1}{x+y}\right)=2(x+y)^2.2\left(\dfrac{1}{x+y}\right)^2=4$

7.(AB) $f(x)=\sin x+\pi\sin x+\displaystyle\int_{-\pi/2}^{\pi/2} t\ f(t)dt$

$f(x)=(1+\pi)\sin x+A$ and $A=\displaystyle\int_{-\pi/2}^{\pi/2} t\ f(t)\ dt=\int_{-\pi/2}^{\pi/2}\big(t\cdot(1+\pi)\sin t+tA\big)dt$

$$A=\left[(1+\pi)\big(\sin t-t\ \cos t\big)+A\frac{t^2}{2}\right]_{-\pi/2}^{\pi/2}$$

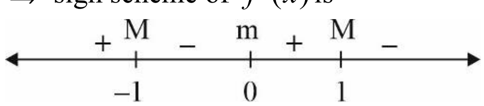
$\Rightarrow \quad A=2(1+\pi)-0+0 \;\Rightarrow\; f(x)=(1+\pi)\sin x+2\pi+2$

Maximum $=M=3\pi+3$ Minimum $=m=\pi+1$

8.(AB) $f'(x)=\dfrac{2x}{(1+x^2)}-\dfrac{2\big(2x\big).\ x^2}{(1+x^2)^3}$

$=\dfrac{2x(1+x^2)-4x^3}{(1+x^2)^3}=\dfrac{2x(1-x^2)}{(1+x^2)^3}$

$\Rightarrow$ sign scheme of $f'(x)$ is



9.(ABC) $F'(x)=f(x).g'(x)+f'(x).g(x)$

$=f'(x).g'(x)\left(\dfrac{f(x)}{f'(x)}+\dfrac{g(x)}{g'(x)}\right)=c\left(\dfrac{f(x)}{f'(x)}+\dfrac{g(x)}{g'(x)}\right)$

and $F''(x)=f(x).g''(x)+f''(x).g(x)+2f'(x).g'(x)$

$\Rightarrow \dfrac{F''(x)}{F(x)}=\dfrac{g''(x)}{g(x)}+\dfrac{f''(x)}{f(x)}+\dfrac{2c}{f(x).g(x)}$

Again $F''(x)=f(x).g''(x)+f''(x).g(x)+2c$

Then $F'''(x)=f(x).g'''(x)+f'(x).g''(x)+f'''(x)g(x)+f''(x).g'(x)+O$

$=f(x).g'''(x)+f'''(x).g(x)+\dfrac{d}{dx}(f'(x).g'(x))$

$F''(x)=f(x).g'''(x)+f'''(x).g(x)$

$\Rightarrow \dfrac{F''(x)}{F(x)}=\dfrac{g'''(x)}{g(x)}+\dfrac{f'''(x)}{f(x)}$

10.(BD) Let $f(x)=x^2+px+q$

$f'(x)=2x+p \Rightarrow f'(-3)=p-6$

$f''(x)=2 \Rightarrow f''(-2)=2$

Now $g(x)=x^2-xf'(-3)+f''(-2)$

$\Rightarrow g(x)=x^2-(p-6)x+2$

$g'(x)=2x-(p-6) \Rightarrow g'(1)=8-p$

$g''(x)=2 \Rightarrow g''(2)=2$

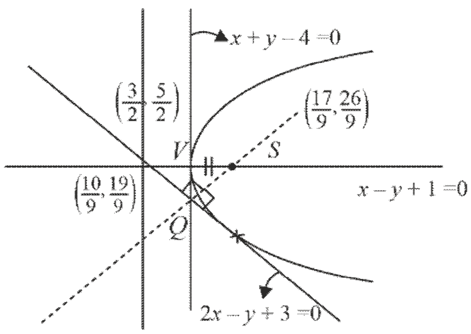
$\Rightarrow f(x)=x^2+xg'(1)+g''(2) = x^2+(8-p)x+2=x^2+px+q$

$\Rightarrow p=4, \; q=2$

$\Rightarrow f(x)=x^2+4x+2$ and $g(x)=x^2+2x+2$

PARAGRAPH FOR Q-11 & 12

11.(C)



Here, vertex $\equiv\left(\frac{3}{2},\frac{5}{2}\right), Q=\left(\frac{1}{3},\frac{11}{3}\right)$.

As the feet of perpendicular from focus on any tangent lies on tangent at vertex, so focus $S\equiv\left(\frac{17}{9},\frac{26}{9}\right)$.

Also, vertex is the midpoint of foot of directrix and focus.

So, foot of directrix $\equiv\left(\frac{10}{9},\frac{19}{9}\right)$

As, directrix is perpendicular to axis, so its equation is $x+y=\lambda$

Also, $\left(\frac{10}{9},\frac{19}{9}\right)$ lies on it, so $x+y=\frac{29}{9}$ or $9x+9y=29$ (equation of directrix)

So, $l=9$ and $m=9$ Hence, $(l+m)=9+9=18$

12. (D) We know that the length of latus rectum of parabola = 4(Distance between vertex and focus)

$$=4\sqrt{\left(\frac{17}{9}-\frac{3}{2}\right)^2+\left(\frac{26}{9}-\frac{5}{2}\right)^2}=4\sqrt{\left(\frac{7}{18}\right)^2+\left(\frac{7}{18}\right)^2}=4\sqrt{2}\times\frac{7}{18}=\frac{14\sqrt{2}}{9}=\frac{p\sqrt{2}}{q}$$

$\therefore p=14$ and $q=9$

Hence, $(p+q)=14+9=23$

SECTION-2

1.(1) Focus $S=(Ae,0)A=3a, B=2a \Rightarrow e=\frac{1}{3}\sqrt{5}$

Point on $\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$ is $(A\cos\theta, B\sin\theta)$

$\Rightarrow$ Equation of tangent $\frac{x}{A}\cos\theta+\frac{y}{B}\sin\theta=1$

$$Bx\cos\theta+Ay\sin\theta-AB=0$$

Now $\perp$ distance from focus $=p=\left|\frac{BAe\cos\theta+0-AB}{\sqrt{B^2\cos^2\theta+A^2\sin^2\theta}}\right|$

$$\Rightarrow \quad p=\left|\frac{AB(1-e\cos\theta)}{\sqrt{A^2(1-e^2)\cos^2\theta+A^2\sin^2\theta}}\right|=\frac{B(1-e\cos\theta)}{\sqrt{1-e^2\cos^2\theta}}$$

$$\Rightarrow \frac{B^2}{p^2}=\frac{1+e\cos\theta}{1-e\cos\theta} \qquad \ldots(1)$$

And the distance between the focus and the point

$$r=\sqrt{(A\cos\theta-Ae)^2+B^2\sin^2\theta}$$

$$=A\sqrt{(\cos^2\theta+e^2-2e\cos\theta)+(1-e^2)\sin^2\theta}=A\sqrt{1+e^2\cos^2\theta-2e\cos\theta}=A(1-e\cos\theta)$$

$$\Rightarrow \quad \frac{A}{r}=\frac{1}{1-e\cos\theta} \qquad \ldots(2)$$

$$\frac{6a}{r}-\frac{4a^2}{p^2}=2\frac{A}{r}-\frac{B^2}{p^2}=\frac{2}{1-e\cos\theta}-\frac{1+e\cos\theta}{1-e\cos\theta}=1$$

2.(2) The four tangents form a rectangle (square) $\Rightarrow m=\pm1$ and equation of tangent $y=mx\pm\sqrt{a^2m^2+b^2}$ and $y=mx\pm r\sqrt{2}\sqrt{1+m^2}$ for common tangent $a^2m^2+b^2=2r(1+m^2)$

$$\Rightarrow a^2+b^2=4r^2$$

Now $\frac{2r_1^2}{r_2^2}=\frac{2(a^2+b^2)}{2(r\sqrt{2})^2}=\frac{a^2+b^2}{2r^2}=2$

3.(120) Both curves are mirror images of each other about $y=x$.

$\Rightarrow$ Shortest distance between the curves is equal to the distance parallel tangents parallel to the line $y=x$.

Let the line $y=x+c$ touches $x^2=y-4$

$\Rightarrow x^2=x+c-4, D=0 \Rightarrow c=-\frac{15}{4}$

$\Rightarrow$ the parallel tangents are $y=x-\frac{15}{4}$ and $y=x+\frac{15}{4}$

$\Rightarrow$ distance between the tangents $=d=\frac{15}{2\sqrt{2}}$

4.(160) $\frac{-b}{2a}=1 \Rightarrow b=-2a$

$\frac{-D}{4a}=2 \Rightarrow \frac{4ac-b^2}{4a}=2 \Rightarrow c=a+2$

Now the product $P=abc=-2a^2(a+2)$

$$P=-2a^3-4a^2\ ;\ \frac{dp}{da}=-6a^2-8a<0 \forall a \in [2,4]$$

$\Rightarrow P$ is a decreasing function of a

$\Rightarrow P(4)=-2(4)^2(4+2)=-192$ (minimum)

$P(2)=-2(2)^2(2+2)=-32$ (maximum)

5.(3) $f(x)=\left[\tan x(\cot x-\{\cot x\})\right]$

$=\left[1-\tan x.\{\cot x\}\right]$

$\begin{cases} 1, & \cot x = \text{integer} \\ 0, & \text{otherwise} \end{cases}$

in $x \in \left[\frac{\pi}{12},\frac{\pi}{4}\right] \to \tan x \le 1$ and $0 \le \{\cot x\} < 1$

So, $f(x)=1 \quad \Rightarrow \quad \{\cot x\}=0 \quad \Rightarrow \quad \cot x = \text{integer}$

$x \in \left[\frac{\pi}{12},\frac{\pi}{4}\right] \Rightarrow \cot x \in \left[1,2+\sqrt{3}\right] \Rightarrow$ 3 integer values of $\cot x = 1,2,3$.

6.(3) $x^3+y^3=r^3 \Rightarrow 3x^2+3y^2\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=-\frac{x^2}{y^2}$

Slope of tangent at $(a,b)=-\frac{a^2}{b^2}$

This tangent meets the curve again

$\Rightarrow$ Slope of tangent at A = slope of chord AB

$$\Rightarrow \quad -\frac{a^2}{b^2}=\frac{d-b}{c-a} \qquad \ldots(1)$$

Given that $a^3+b^3=r^3$ and $c^3+d^3=r^3$

$$\Rightarrow \quad c^3-a^3+d^3-b^3=0$$

$$\Rightarrow \quad \frac{d-b}{c-a}=-\frac{c^2+a^2+ac}{d^2+b^2+db} \qquad \ldots(2)$$

Now $\quad \frac{a^2}{b^2}=\frac{c^2+a^2+ac}{d^2+b^2+db}$

$$a^2d^2+a^2b^2+a^2bd=b^2c^2+b^2a^2+acb^2$$

$$a^2d^2-b^2c^2=acb^2-a^2bd$$

$$(ad-bc)(ad+bc)=ab(bc-ad)$$

$$ad+bc=-ab$$

$$\frac{d}{b}+\frac{c}{a}=-1\ ;\ \frac{2d}{b}+\frac{2c}{a}+5=3$$